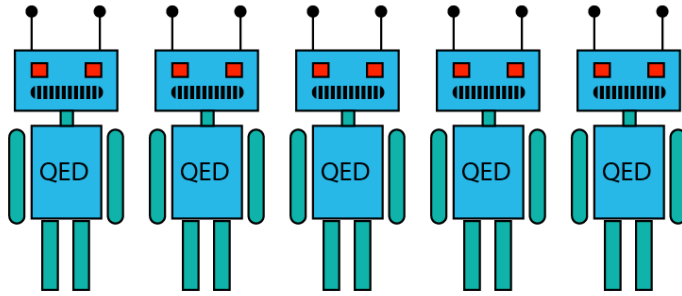


MATHS CHALLENGE 2010

SOLUTIONS – PART I

EASY 1

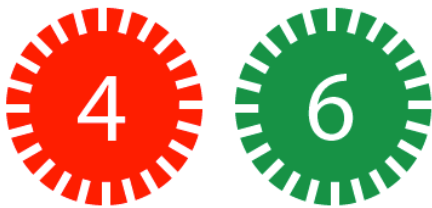
If five robots can make five new robots in five hours, how long would it take a hundred robots to make a hundred new robots?



ANSWER: In five hours, each of the five robots makes one robot. That is, a single robot takes five hours to make a robot. So, in five hours a hundred robots can make a hundred robots.

EASY 2

Two poker chips are pictured below. The two visible sides add up to 10. Flipping the chips in all possible ways, you get the sums 7, 8, 9, and 10. What are the numbers on the other side of the chips?



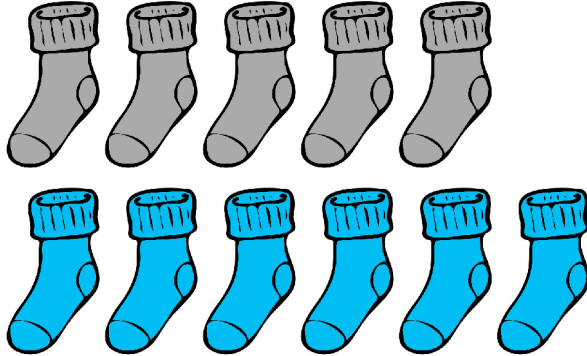
ANSWER: One of the sums is 9, which gives us three scenarios:

- The green 6 is part of the 9. Then the hidden side of the red chip is 3, and it follows that the hidden side of the green chip is 4.
- The red 4 is part of the 9. Then the hidden side of the green chip is 5, and it follows that the hidden side of the red chip is 2.
- The sum of the two hidden sides is 9.

In fact, the third scenario is impossible. The two remaining sums are 8 and 7, which means the hidden side of the red chip would be at most 2, and the hidden side of the green chip would be at most 4. So, the two hidden sides cannot add to 9.

EASY 3

A drawer full of socks contains 5 black socks and 6 blue socks. You take socks out of the drawer without looking. How many socks must you take out to be sure of having a matching pair? What if you want to be sure of having a black pair?



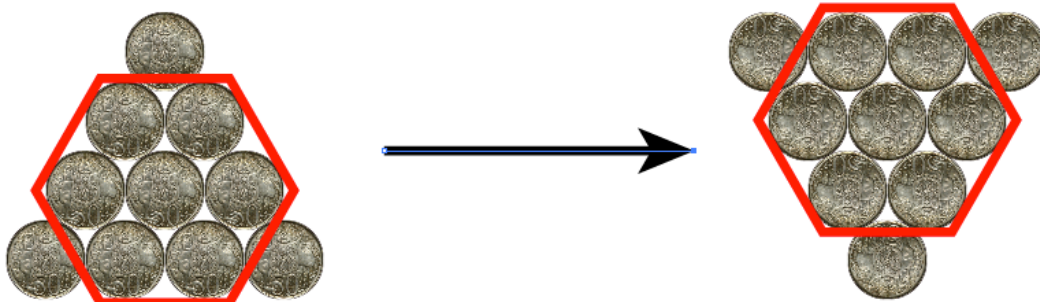
ANSWER: Three socks must give a matching pair, since either two of the socks are blue, or two are black. Eight socks are needed to ensure a black pair, since the first six you choose might all be blue.

EASY 4

The triangle of coins below is pointing upwards. By moving only three coins, arrange for the triangle to point downwards.

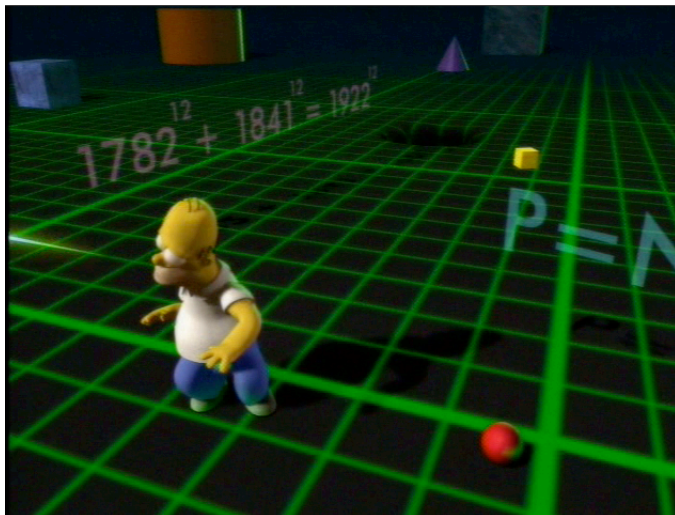


ANSWER: Keep the hexagon in the middle, and move the three corner coins.



EASY 5

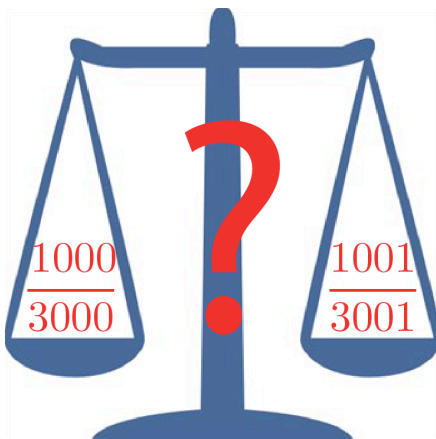
Three-dimensional Homer stumbles across the equation $1782^{12} + 1841^{12} = 1922^{12}$. Is the equation true or false? (Put that calculator down!)



ANSWER: The left side of the equation is EVEN + ODD = ODD, and the right side of the equation is EVEN. So, the equation must be false.

EASY 6

Which is larger, $1000/3000$ or $1001/3001$? (Put that calculator down!)



ANSWER: Since $1000/3000 = 1/3 = 1001/3003$, the fraction $1001/3001$ must be larger.

EASY 7

Julie takes a third of the jellybeans from a large jar. Then Tony takes a third of the remaining jellybeans from the jar. Finally, Bob takes a third of the remaining jellybeans, leaving 40 jellybeans in the jar. How many jellybeans were there to begin with?



ANSWER: Bob left 40 beans, which was $\frac{2}{3}$ of the beans. So, at that stage there were 60 beans. That means 60 beans amounted to $\frac{2}{3}$ of the beans when Tony took his beans, so there must have been 90 beans at that stage. Finally, those 90 beans were $\frac{2}{3}$ of the beans when Julie took hers, so there must have initially been 135 beans.

EASY 8

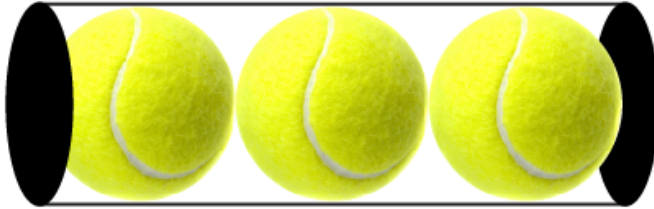
A bat and a ball together cost \$110. The bat costs \$100 more than the ball. How much does the ball cost?



ANSWER: The ball cost \$5 and the bat cost \$105.

EASY 9

Three tennis balls fit exactly inside a cylindrical can. Which is greater, the circumference of the can or the length of the can? Which is greater, the combined area of the three balls, or the area of the cylinder (excluding the base and the lid)?



ANSWER: Call the radius of the balls R . So, the length of the can is $6R$ and the circumference of the can (equals the length of the equators of the balls) is $2\pi R$. Since $\pi > 3$, the circumference is greater than the length. Also, the area of the cylinder is $2\pi R \times 6R = 12\pi R^2$. The area of each ball is $4\pi R^2$, so the combined area of the balls is the same as the area of the cylinders.

EASY 10

Renaë walks into an electronics store to purchase a \$1000 television, and she has three coupons. One coupon entitles Renaë to a 50% discount, the second to a 30% discount, and the third to a 20% discount. David the salesman agrees to let Renaë use all three coupons at once. How much does Renaë pay for the television?



ANSWER: The 50% discount coupon means that Renaë would pay only \$500. Then, the 30% discount coupon means that Renaë only has to pay 70% of that \$500, and so the television would cost \$350. Finally, the 20% discount coupon applied to that \$350, lowers the price another \$70, and so Renaë only has to pay \$280.

Note that it doesn't matter in which order Renaë applies the coupons: in any case we're multiplying \$1000 by $0.5 \times 0.7 \times 0.2 = 0.28$. Also, if Renaë confronts a *really* gullible salesman, she might actually succeed in obtaining a $50\% + 30\% + 20\% = 100\%$ discount!